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A Novel Range-Free Node Localization Method for Wireless Sensor Networks

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***Abstract*—Solving the nonconvex and non-differentiable objective function of traditional range-free node localization methods will result in low localization accuracy or high computational complexity. To this end, a novel iterative localization algorithm called CVX-DV-hop is proposed in this paper. It first performs matrix transformation to reformulate the original optimization problem into one with a convex function and nonconvex constraints. And then, the first-order Taylor expansions are employed to tighten the nonconvex constraints into linear inequality constraints. Finally, a successive convex approximation method is designed to iteratively solve the optimization problem. Simulation results show that the proposed algorithm has higher localization accuracy and lower computational burden than those of the particle swarm optimization algorithm.**

***Index Terms*—localization, optimization, range-free, successive convex approximation, wireless sensor networks (WSN).**

I. Introduction

A. Research Background and State of The Art

WIRELESS sensor networks (WSN) are composed of energy-limited sensor nodes with characteristics such as self-organization, low cost, dynamic topology [1]. Recently, developments of microelectronics and embedded technology make possible the multi-function, high integration, and miniaturization of sensor nodes, leading to its wide application in environmental monitoring, fire detection, industrial monitoring, military reconnaissance and so on [2]. In WSN, the location label of nodes is important to processing data collected by nodes [3]. The global positioning system (GPS) may be an option to obtain the location label of nodes; however, all nodes in WSN equipped with GPS may increase cost and energy consumption of WSN. Thus, node localization methods without GPS assistance have attracted great attention of the research community over the last decade.

So far, node localization schemes can be divided into two categories: range-based and range-free, according to whether the distance measure is involved between nodes. When each node in WSN has a distance measurement device, one may employ the range-based scheme to obtain distance of nodes. On the other hand, supposing each node in WSN has no distance measure device, the range-free method may be employed to estimate distance between nodes according to network connectivity [4]. Given the characteristics of low cost and convenient realization, range-free methods, such as DV-hop [5], APIT [6], MDS [7] and the centroid algorithms [8], have been widely used in lots of WSN systems. Among them, the DV-hop algorithm is the most classic one, due to its easy implementation. However, since it employs multi-hop distance to approximate straight-line distance, the algorithm typically has high localization error.

To reduce the localization error of the DV-hop algorithm, many improved methods have been proposed. For instance, in the CC-DV-hop algorithm [9], some reference nodes are selected to obtain the average hop distance according to a distance error factor, leading to an increase in node localization accuracy. However, due to its sensitivity to selected reference nodes, CC-DV-hop experiences poor robustness. To enhance the robustness, a new range-free node localization model without need for selecting reference nodes is proposed in [10], where the distance items are resolved from the least-square operator. However, since the objective function in the model is nonconvex and non-differentiable, directly employing the gradient-descent method to tackle it is impracticable. To cope with the problem, the particle swarm optimization (PSO) algorithm is further introduced in [10], which is referred to as PSO-DV-hop in this paper. As it is known, the node localization accuracy of PSO-DV-hop is determined by the number of particles and iterations, and its computational complexity may increase exponentially and linearly with the number of particles and iterations, respectively. Additionally, some bionic optimization algorithms including the genetic algorithm [11], evolutionary algorithm [12] and differential evolution [13] have also been introduced to localize WSN node. However, these meta heuristic methods are usually based on random searching in the feasible set of the optimization problem, leading to irreconcilable tradeoffs between computational complexity and localization accuracy.

In this paper, we present an iterative algorithm to tackle the nonconvex and non-differentiable problem in range-free localization of WSN nodes. Compared with the PSO-DV-hop algorithm, the proposed algorithm can not only enhance the localization accuracy but also reduce the computational complexity. In addition, although the computational complexity of proposed algorithm is higher than that of the DV-hop and CC-DV-hop algorithms, the localization accuracy of the proposed algorithm is significantly improved.

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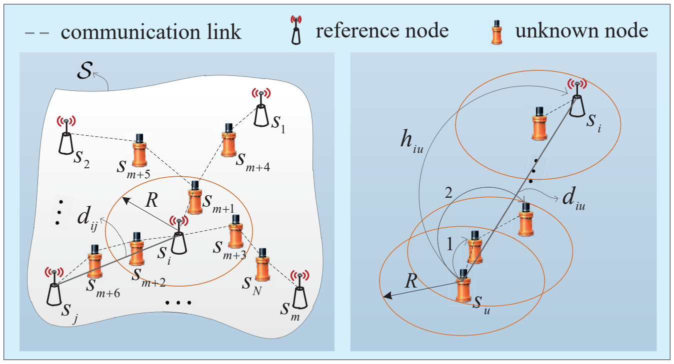

Fig. 1. Network model of cooperative localization.

B. Contributions and Paper Organization

1) Through matrix transformation, the mathematical model of the traditional range-free node localization is transformed into an optimization problem with a convex objective function and non-convex constraints.
2) The nonconvex constraints is tighten by using its first-order Taylor expansions firstly, and then a successive convex approximation algorithm is proposed to iteratively solve the transformed optimization problem.
3) By searching in the feasible set with the gradient-descent method instead of in a random manner, the proposed algorithm exhibits better localization accuracy and lower computation burden compared to the PSO-DV-hop method.

The rest of this paper is organized as follows. System model and problem statement of the WSN localization are described in Section II. The principle of the proposed algorithm is introduced in detail in Section III. In Section IV, the effectiveness of the proposed algorithm are demonstrated through simulation experiments, followed by conclusions in Section V.

II. System Model and Problem Statement

A hybrid wireless sensor network is composed of reference and unknown nodes. While a reference node equipped with GPS can directly obtain location information and broadcast it to neighboring nodes, an unknown node is not equipped with GPS, and thus its location is not readily available. However, in this case the location of reference nodes and topology of WSN can be used to estimate the location of unknown nodes.

For the sake of simplicity, we consider a two-dimensional model; however, our method can be easily extended to three-dimensional scenarios. In Fig. 1, a set of N sensor nodes $\mathcal{S} = \{s_1, \cdots, s_N\}$ are randomly deployed in the monitoring area. Assuming the first m sensor nodes $s_i\,(i = 1, \cdots, m)$ of the set $\mathcal{S}$ are reference nodes, and thus $s_u\,(u = m+1, \cdots, N)$ denotes the unknown nodes, we define the proportion of reference nodes as N_r, namely $N_r = m/N$. Communication radius of each node is set by R, which indicates that communication link between two nodes exists at the distance of these nodes less than certain R. In addition, the location coordinates of the reference node s_i and unknown node s_u are represented by $\boldsymbol{x}_i = [x_i, y_i]^{\mathrm{T}}$ and $\boldsymbol{x}_u = [x_u, y_u]^{\mathrm{T}}$, respectively, where $(\cdot)^{\mathrm{T}}$ indicates transpose. Also, d_{iu} and h_{iu} are the Euclidean distance and the number of hops between the unknown node s_u and the reference node s_i, respectively.

The DV-hop algorithm is proposed by Niculescu [5], in which the distance estimate $\hat{d}_{iu}$ between the unknown node and the reference node is computed according to the network connectivity. It can be described as

$$\hat{d}_{iu} = \overline{d}_i \times h_{iu} \tag{1}$$

where $\overline{d}_i$ is the average hop distance related to the reference node s_i, i.e.,

$$\overline{d}_i = \sum_{j\neq i} d_{ij} \bigg/ \sum_{j\neq i} h_{ij}, \quad i, j = 1, \cdots, m. \tag{2}$$

where d_{ij} and h_{ij} represents the distance and the number of hops between the reference nodes s_i and s_j, respectively.

Then, employing the multilateral localization principle based on least squares, we obtain the coordinate estimate of the unknown node s_u as [9]

$$\hat{\boldsymbol{x}}_u = (\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A})^{-1}\boldsymbol{A}^{\mathrm{T}}\boldsymbol{b} \tag{3}$$

where,

$$\boldsymbol{A} = 2\begin{bmatrix} (x_1 - x_m) & (y_1 - y_m) \\ (x_2 - x_m) & (y_2 - y_m) \\ \vdots & \vdots \\ (x_{m-1} - x_m) & (y_{m-1} - y_m) \end{bmatrix},$$

$$\boldsymbol{b} = \begin{bmatrix} x_1^2 - x_m^2 + y_1^2 - y_m^2 + \hat{d}_{mu}^2 - \hat{d}_{1u}^2 \\ x_2^2 - x_m^2 + y_2^2 - y_m^2 + \hat{d}_{mu}^2 - \hat{d}_{2u}^2 \\ \vdots \\ x_{m-1}^2 - x_m^2 + y_{m-1}^2 - y_m^2 + \hat{d}_{mu}^2 - \hat{d}_{(m-1)u}^2 \end{bmatrix}$$

It can be seen that $\hat{d}_{iu}$ in (1) is roughly estimated by multiplying the average hop distance and the number of hops, and then linear least squares in (3) is used to estimate the nodes location. Since employing the linear model, the DV-hop algorithm typically results in large localization error. To enhance the localization accuracy, the PSO-DV-hop algorithm based on an accurate nonlinear model is proposed to estimate the location of nodes, which formulates the problem alternatively as

$$\min_{\boldsymbol{x}_u} \sum_{i=1}^{m} \left| \|\boldsymbol{x}_u - \boldsymbol{x}_i\|_2 - \hat{d}_{iu} \right| \tag{4}$$

where $\|\cdot\|_2$ and $|\cdot|$ denote the ℓ_2 norm and the absolute value, respectively.

III. Proposed Localization Algorithm

A. CVX-DV-hop Algorithm

Obviously, the problem in (4) is non-convex and non-differentiable [14]. As such, we propose a new improved algorithm here called CVX-DV-hop. To make the solution of (4) easier, we first introduce auxiliary variables $p_i, (i =$

TABLE I
PSEUDO-CODE OF PROPOSED ALGORITHM

CVX-DV-hop Algorithm:
Input: location coordinate vectors $\{\boldsymbol{x}_i\}$ of the reference nodes $\{s_i\}\,(i=1,\cdots,m)$;
Output: location coordinate vector $\hat{\boldsymbol{x}}_u$ of the unknown node s_u;
1. Computing the distance $\hat{d}_{iu}$ according to (1);
2. Computing the initial location $\boldsymbol{x}_u^0$ based on (3) and setting an accuracy threshold ϵ;
3. **Repeat:**
4. Solving the problem (7) to obtain the optimal solution $\boldsymbol{x}_u^{k+1}$;
5. $\boldsymbol{x}_u^k \Leftarrow \boldsymbol{x}_u^{k+1}$;
6. **Until:** $\left\|\boldsymbol{x}_u^{k+1}-\boldsymbol{x}_u^k\right\|_2 < \epsilon$

$1,2,\cdots,m)$ to (4), leading to an optimization problem reformulated as

$$\min_{\boldsymbol{p},\boldsymbol{x}_u} \quad \mathbf{1}^{\mathrm{T}}\cdot\boldsymbol{p} \tag{5a}$$

$$\text{s.t.} \quad \|\boldsymbol{x}_u-\boldsymbol{x}_i\|_2-\hat{d}_{iu} \geq -p_i, i=1,2,\cdots,m. \tag{5b}$$

$$\|\boldsymbol{x}_u-\boldsymbol{x}_i\|_2-\hat{d}_{iu} \leq p_i, i=1,2,\cdots,m. \tag{5c}$$

where $\mathbf{1}$ is the all-one column vector of length m, and $\boldsymbol{p}=[p_1,p_2,\cdots,p_m]^{\mathrm{T}}$.

Given the fact that the objective function (5a) is convex and differentiable, and the set of inequalities (5c) are convex as well, the nonconvexity lies only in the inequality constraints (5b). Thus, we now focus on a convex approximation of (5b). To this end, since the left-hand side of the inequalities (5b) are convex functions, we can tighten the constraints by using their global under-estimator. Specifically, the first-order Taylor lower bound of the left-hand side of the inequalities (5b) can be expressed as

$$\|\boldsymbol{x}_u-\boldsymbol{x}_i\|_2-\hat{d}_{iu} \geq \left\|\boldsymbol{x}_u^0-\boldsymbol{x}_i\right\|_2-\hat{d}_{iu}+\frac{\left(\boldsymbol{x}_u^0-\boldsymbol{x}_i\right)^{\mathrm{T}}}{\|\boldsymbol{x}_u^0-\boldsymbol{x}_i\|_2}\left(\boldsymbol{x}_u-\boldsymbol{x}_u^0\right) \tag{6}$$

where $\boldsymbol{x}_u^0$ is the initial location of the node s_u. It can be seen that the non-convex constraints (5b) have been tightened into affine inequality constraints. Correspondingly, by the above approximation, the optimization problem (5) has been transformed into the following form, i.e.,

$$\begin{aligned} &\min_{\boldsymbol{p},\boldsymbol{x}_u} \ \mathbf{1}^{\mathrm{T}}\cdot\boldsymbol{p} \\ &\text{s.t.} \ \left\|\boldsymbol{x}_u^0-\boldsymbol{x}_i\right\|_2-\hat{d}_{iu}+\tfrac{\left(\boldsymbol{x}_u^0-\boldsymbol{x}_i\right)^{\mathrm{T}}}{\|\boldsymbol{x}_u^0-\boldsymbol{x}_i\|_2}\left(\boldsymbol{x}_u-\boldsymbol{x}_u^0\right) \geq -p_i, \\ &\quad i=1,2,\cdots,m. \\ &\quad \|\boldsymbol{x}_u-\boldsymbol{x}_i\|_2-\hat{d}_{iu} \leq p_i, i=1,2,\cdots,m. \end{aligned} \tag{7}$$

According to the above analysis, the non-convex and non-differentiable problem (4) has been tightened to a convex optimization problem (7). Therefore, given an initial location $\boldsymbol{x}_u^0$, the optimal $\boldsymbol{x}_u^1$ can be easily obtained by using the gradient descent method. Furthermore, the optimization problem (7) can be solved in an iterative way to obtain the optimal solution that satisfies a certain accuracy threshold ϵ. More details related to the proposed algorithm are shown in Table I.

TABLE II
COMPLEXITY COMPARISON OF DIFFERENT LOCALIZATION ALGORITHMS

Algorithm	Complexity in general	Complexity in the simulation scenes
DV-hop	$\mathcal{O}\left(m^3\right)$	9.56×10^6
CC-DV-hop	$\mathcal{O}\left(m^3\right)$	9.56×10^6
PSO-DV-hop	$\mathcal{O}\left(N_{iter}\cdot4\cdot m\cdot {N_p}^2\right)$	1.70×10^9
CVX-DV-hop	$\mathcal{O}\left(\sqrt{3m}\cdot m^3\cdot\ln(1/\epsilon)\right)$	7.67×10^8

TABLE III
SIMULATION PARAMETERS

Parameter	Symbol	Value
Area of simulation (m^2)	$L\times L$	100×100
Total number of nodes	N	150
Proportion of reference nodes	N_r	30%
Number of reference nodes	m	45
Number of unknown nodes	$N-m$	105
Communication radius (m)	R	25
Threshold	ϵ	10^{-3}
Simulation times	M_c	100

B. Complexity Analysis

The total complexity of the proposed CVX-DV-hop algorithm is mainly determined by the number of iterations and the computational complexity of each iteration. Since the problem (7) has $m+2$ optimized variable and includes m linear inequality constraints and m second-order cone constraints, similar to [15], the complexity of solving (7) is on the order of $\mathcal{O}\left(m^3\right)$. Moreover, the optimization iterations needed to satisfy the accuracy threshold ϵ is about $\sqrt{3m}\cdot\ln(1/\epsilon)$ [15]. Therefore, the total computational complexity of the proposed algorithm is roughly $\mathcal{O}\left(\sqrt{3m}\cdot m^3\cdot\ln(1/\epsilon)\right)$. For comparison, the computational complexities of DV-hop [5], CC-DV-hop [9] and PSO-DV-hop [10] are collected and shown in the second column of Table II, where N_p and N_{iter} are the number of populations and iterations of the PSO-DV-hop algorithm, respectively.

IV. SIMULATION RESULTS AND ANALYSIS

To evaluate the accuracy of the proposed algorithm, we here adopt three performance metrics: root mean square error (RMSE), average localization error (ALE) and localization error variance (LEV) [10], which are defined as

$$\mathrm{RMSE}=\sqrt{\frac{1}{M_c}\sum\nolimits_{t=1}^{M_c}\left\|\hat{\boldsymbol{x}}_u^t-\boldsymbol{x}_u^{true}\right\|_2^2} \tag{8}$$

$$\mathrm{ALE}=\frac{\sum_{u=m+1}^{N}\sqrt{\|\hat{\boldsymbol{x}}_u-\boldsymbol{x}_u^{true}\|_2^2}}{(N-m)\times R} \tag{9}$$

$$\mathrm{LEV}=\sqrt{\frac{\sum_{u=m+1}^{N}\left(\|\hat{\boldsymbol{x}}_u-\boldsymbol{x}_u^{true}\|_2-\mathrm{ALE}\times R\right)^2}{(N-m)\times R^2}} \tag{10}$$

where M_c, $\hat{\boldsymbol{x}}_u$ and $\boldsymbol{x}_u^{true}$ represent the number of simulation experiments, the estimated and true location coordinates of the unknown node s_u, respectively.

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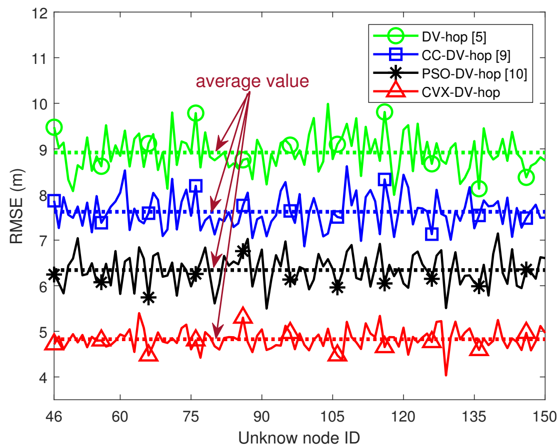

Fig. 2. RMSE performance of unknown nodes localization.

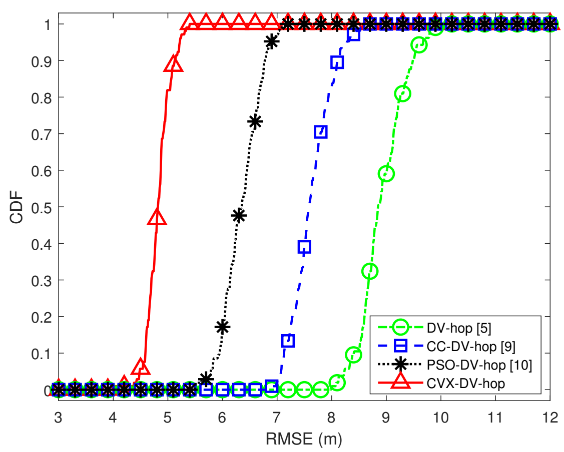

Fig. 3. CDF of localization error of unknown nodes.

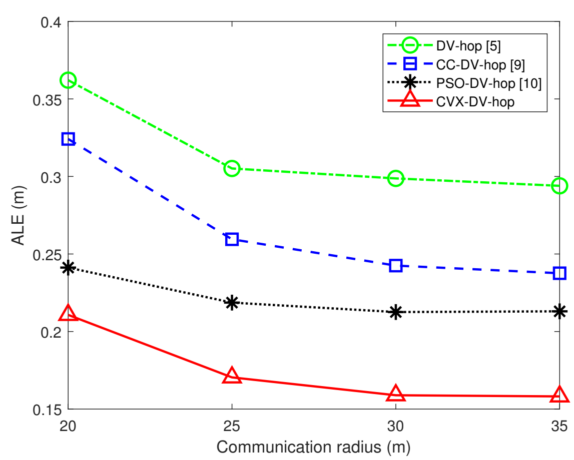

Fig. 4. ALE performance versus the communication radius.

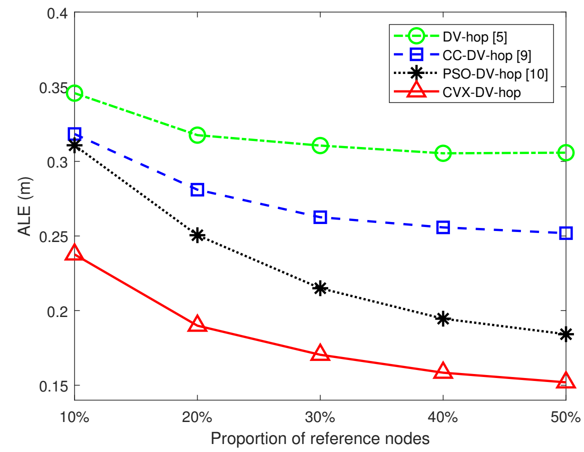

Fig. 5. ALE performance versus the proportion of reference nodes.

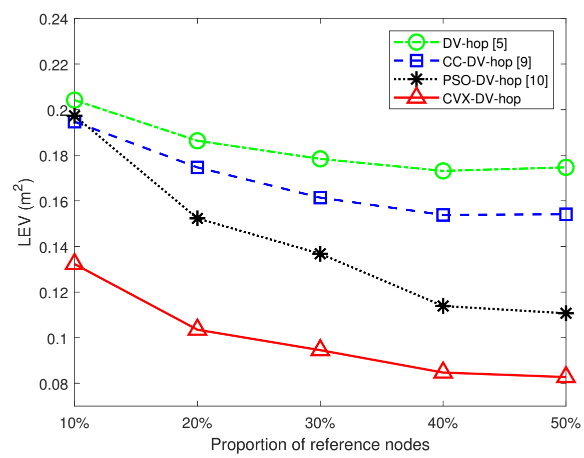

Fig. 6. LEV performance versus the proportion of reference nodes.

The simulation parameters are listed in Table III, based on which the RMSE performance of the each unknown node is shown in Fig. 2. It can be seen that the four algorithms, sorted in the decreasing localization accuracy, are CVX-DV-hop, PSO-DV-hop, CC-DV-hop and DV-hop. Quantitatively, by averaging RMSE among all the unknown nodes, we can have that the accuracy of the proposed algorithm is improved by 23.92%, 36.67% and 45.90%, compared to the PSO-DV-hop, CC-DV-hop and DV-hop algorithms, respectively. In addition, the cumulative distribution function (CDF) of the localization error of the four algorithms is shown in Fig. 3. As expected, the CDF curve of the CVX-DV-hop reaches the maximum first, which demonstrates that the proposed algorithm can achieve the highest localization accuracy.

The ALE performances versus the communication radius and proportion of reference nodes are plotted in Figs. 4 and 5, respectively. It can be seen that the ALE of the proposed algorithm is the lowest. Moreover, the ALE curves of all four algorithms show a decreasing trend as the proportion of reference nodes increases. The reason behind this observation is that increasing the communication radius R or the proportion of reference nodes N_r can enhance average connectivity of WSN. Accordingly, a more accuracy estimate $\hat{d}_{iu}$ can be achieved, which finally results in an improvement in node localization accuracy. The LEV performance versus the proportion of reference nodes is plotted in Fig. 6. As expected, the proposed algorithm has the best LEV performance in all cases which indicates the accuracy of proposed algorithm is the highest in the four algorithms.

In addition, we present the computational complexity of the four algorithms in the third column of Table II, which corresponds to the simulation parameters in Table III. It is can be found that computational complexity of CVX-DV-hop is about 45% of that of PSO-DV-hop. Indeed, computational complexity of proposed algorithm are higher than that of DV-hop and CC-DV-hop algorithm. However, as described in Figs. 4, 5 and 6, the localization accuracy of the proposed algorithm is higher than that of the DV-hop and CC-DV-hop algorithms.

V. Conclusion

In this paper, a novel range-free node localization algorithm is presented. We employ matrix transformation and the first-order Taylor expansion to convert the conventional non-convex and non-differentiable problem into an iterative convex optimization problem, by which high-precision and low-complexity localization of wireless sensor nodes can be achieved. In our future work, it would be interesting to explore and further narrow the performance gap between the approximate solution and the optimal solution of the original optimization problem in (4).

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